1. INTRODUCTION

Quantum entanglement is the key ingredient in and exhibits the fundamental feature of quantum mechanics, and it has played a significant role in quantum information since the Einstein–Podolsky–Rosen (EPR) paradox was proposed by Einstein et al. [1]. With the development of quantum information, multipartite continuous-variable (CV) entanglement has been recognized as the essential resource for high-capability multipartite quantum communications, such as quantum teleportation networks [2], quantum teleporting [3,4], and quantum-controlled dense coding [5,6]. In recent years, considerable interest has been shown for generating multipartite CV entanglement sources. So far, the commonly used method in the generation of multipartite CV entanglement is to use nonlinear optical processes that can produce multicolor entangled beams. Bipartite entanglement has been theoretically and experimentally realized in below-threshold optical parametric oscillators (OPOs) [11–13]. The complementary process of second-harmonic generation (SHG) can also generate bright bipartite entanglement [14]. For tripartite entanglement, Nussenzveig’s group has theoretically [15] and experimentally [16,17] shown that an above-threshold OPO can produce three-color CV entanglement. Other methods that generate multipartite entangled beams—by using cascaded nonlinear or concurrent nonlinear processes [18–26]—have also been widely discussed.

The system consisting of two coupled nonlinear waveguides named the nonlinear optical coupler or quantum optical dimer, with a linear coupling between them achieved by evanescent overlapping waves, has been theoretically investigated [27,28]. Herec et al. studied the nonlinear optical coupler that can generate bipartite entanglement from the coupled spontaneous parametric downconversion (PDC) in the traveling-wave configuration [29]. Bache et al. theoretically investigated the quantum optical dimer using coupled SHGs in a Fabry–Perot cavity and analyzed the intensity correlations as well as the phase-dependent correlation features [30]. Later, Olsen’s group theoretically discussed the related device as mentioned in [30] for different circumstances, such as in the below- or above-threshold coupled intracavity optical PDCs [31,32], SHGs [33], and Kerr nonlinear coupler [34], which can generate robust bipartite CV entangled beams and be used to demonstrate the EPR paradox. Recently, Guo et al. [35] have theoretically shown that the coupled PDC in an optical cavity could produce bright quadrpartite CV entangled beams.

Compared with the downconversion process, its complementary process of SHG has some advantages; for example, the possibility of using frequency doubling may be convenient for reaching spectral regions that are not readily accessible by downconversion, and the generated second-harmonic (SH) fields have macroscopic intensities as soon as the input fundamental waves (FWs) are turned on. © 2013 Optical Society of America
entanglement by the coupled SHG processes in an optical cavity. We theoretically demonstrate the CV entanglement among the two FW and two SH beams according to the inseparability criterion for multipartite CV entanglement as proposed by van Loock and Furusawa [36].

2. SCHEME HAMILTONIAN AND THE EQUATIONS OF MOTION

The scheme is schematically depicted in Fig. 1. The system consists of two parallel adjacent nonlinear waveguides and an optical cavity. The two waveguides are fabricated in a periodically poled nonlinear optical crystal. Efficient SHG takes place in each waveguide. These two waveguides are numbered as 1 and 2, respectively. M1 and M2 are the coupler mirrors. The linear coupling between these waveguides is realized by the evanescent overlaps of the intracavity modes within the nonlinear media. Two spatially separated FWS, $a_{1}^{\text{in}}$ and $a_{2}^{\text{in}}$ (with the frequency of $\omega$), are incident upon nonlinear waveguides 1 and 2 through the coupler mirror M1. $b_{1}$ and $b_{2}$ (with the frequency of $2\omega$) are the SHs generated by the SHG processes. Here, we assume that the two SHG processes are equal and all the modes inside the waveguides are perfectly phase matched using the quasi-phase-matching (QPM) technique.

The effective Hamiltonian for this system can be written as

$$\hat{H}_{\text{tl}} = \hat{H}_{\text{sys}} + \hat{H}_{\text{couple}} + \hat{H}_{\text{bath}}.$$  \hspace{1cm} (1)

The interaction Hamiltonian for this system is given by

$$\hat{H}_{\text{sys}} = i\hbar \frac{\kappa}{2} [\hat{a}_{1}^{\dagger}\hat{a}_{1}\hat{b}_{2} - \hat{a}_{2}^{\dagger}\hat{b}_{2}\hat{a}_{1}^\dagger - \hat{a}_{2}^{\dagger}\hat{b}_{2}\hat{a}_{1}^\dagger + \hat{a}_{1}^{\dagger}\hat{b}_{2}\hat{a}_{2}^\dagger] - \hbar [\delta_{a}(\hat{a}_{1}^{\dagger}\hat{a}_{1} + \hat{a}_{2}^{\dagger}\hat{a}_{2}) + \delta_{b}(\hat{b}_{1}^{\dagger}\hat{b}_{1} + \hat{b}_{2}^{\dagger}\hat{b}_{2})],$$  \hspace{1cm} (2)

where the first part of the Hamiltonian stands for the SHG processes, and the second part is the free Hamiltonian of the cavity modes for the FW and SH fields. $\kappa$ is the dimensionless nonlinear coupling coefficient, which is related to the nonlinear susceptibility and structure parameters of the periodically poled nonlinear optical crystal and is taken as real without loss of generality [9], $\delta_{a} = \omega - \omega_{a}^{\text{cw}}$, and $\delta_{b} = 2\omega - \omega_{b}^{\text{cw}}$ are the cavity frequency detunings from their respective resonances.

The coupling Hamiltonian has the form

$$\hat{H}_{\text{couple}} = \hbar J_{a}[\hat{a}_{1}^{\dagger}\hat{a}_{2} + \hat{a}_{1}\hat{a}_{2}^{\dagger}] + \hbar J_{b}[\hat{b}_{1}^{\dagger}\hat{b}_{2} + \hat{b}_{1}\hat{b}_{2}^{\dagger}],$$  \hspace{1cm} (3)

where $J_{a}$ and $J_{b}$ are the linear coupling parameters at two frequencies of the two nonlinear waveguides. Typically the lower-frequency ($\omega$) coupling parameter $J_{a}$ is about 50 times larger than the higher-frequency ($2\omega$) coupling parameter $J_{b}$, as described in [20].

The cavity dampings are described by

$$\hat{H}_{\text{bath}} = \hbar \sum_{n=1}^{2} [\hat{f}_{n}^{a}\hat{a}_{n}^{\dagger} + \hat{f}_{n}^{a}\hat{a}_{n}] + h.c.,$$  \hspace{1cm} (4)

where $\hat{f}_{n}^{a}$ and $\hat{f}_{n}^{b}$ are the bath operators at the two frequencies, respectively.

Following the input–output formalism developed by Collett and Gardiner [37,38], we can obtain the quantum Langevin equations of motion for the four intracavity modes in the Heisenberg picture as [37]

$$\tau \frac{d\hat{a}_{1}}{dt} = (-\gamma_{a} + i\Delta_{a})\hat{a}_{1} + \kappa\hat{b}_{1} - iJ_{a}\hat{a}_{2} + \sqrt{2}\gamma_{a}\hat{a}_{1}^{\dagger},$$

$$\tau \frac{d\hat{a}_{2}}{dt} = (-\gamma_{a} + i\Delta_{a})\hat{a}_{2} + \kappa\hat{b}_{2} - iJ_{a}\hat{a}_{1} + \sqrt{2}\gamma_{a}\hat{a}_{2}^{\dagger},$$

$$\tau \frac{d\hat{b}_{1}}{dt} = (-\gamma_{b} + i\Delta_{b})\hat{b}_{1} - \frac{\kappa}{2}\hat{a}_{1}^{\dagger} - iJ_{b}\hat{b}_{2} + \sqrt{2}\gamma_{b}\hat{b}_{1}^{\dagger},$$

$$\tau \frac{d\hat{b}_{2}}{dt} = (-\gamma_{b} + i\Delta_{b})\hat{b}_{2} - \frac{\kappa}{2}\hat{a}_{2}^{\dagger} - iJ_{b}\hat{b}_{1} + \sqrt{2}\gamma_{b}\hat{b}_{2}^{\dagger}. \hspace{1cm} (5)$$

$\tau$ is the cavity round-trip time, which is assumed to be the same for all cavity modes. $\hat{a}_{1}^{\text{in}}(k = 1,2)$ and $\hat{b}_{2}^{\text{in}}$ are the input field operators of the cavity. The frequency detunings are described by the dimensionless variables $\Delta_{a} = \delta_{a}\tau$ and $\Delta_{b} = \delta_{b}\tau$. $\gamma_{a}$ and $\gamma_{b}$ stand for the loss coefficients of the fundamental and SH fields. For simplicity, we assume the loss coefficients to be $\gamma_{a1} = \gamma_{a2} = \gamma_{a}$, $\gamma_{b1} = \gamma_{b2} = \gamma_{b}$, which are related to the amplitude reflection coefficients $r_{j}(j = a,b)$ and the amplitude transmission coefficients $t_{j}$ approximately as $r_{j} = 1 - t_{j}$ and $t_{j} = \sqrt{2}\gamma_{j}$.

3. STATIONARY SOLUTIONS AND QUANTUM FLUCTUATIONS OF THE OUTPUT MODES

By first neglecting all the correlations and fluctuations and replacing all the operators with their mean values, we can obtain a new set of steady-state equations when setting the left side of Eqs. (5) to be zero [39,40], i.e.,

$$(-\gamma_{a} + i\Delta_{a})\hat{a}_{1} + \kappa\hat{a}_{2} - iJ_{a}\hat{a}_{2} + \sqrt{2}\gamma_{a}\hat{a}_{1}^{\dagger} = 0,$$

$$(-\gamma_{a} + i\Delta_{a})\hat{a}_{2} + \kappa\hat{a}_{1} - iJ_{a}\hat{a}_{1} + \sqrt{2}\gamma_{a}\hat{a}_{2}^{\dagger} = 0,$$

$$(-\gamma_{b} + i\Delta_{b})\hat{b}_{1} - \frac{\kappa}{2}\hat{a}_{1}^{\dagger} - iJ_{b}\hat{b}_{2} + \sqrt{2}\gamma_{b}\hat{b}_{1}^{\dagger} = 0,$$

$$(-\gamma_{b} + i\Delta_{b})\hat{b}_{2} - \frac{\kappa}{2}\hat{a}_{2}^{\dagger} - iJ_{b}\hat{b}_{1} + \sqrt{2}\gamma_{b}\hat{b}_{2}^{\dagger} = 0. \hspace{1cm} (6)$$

where $\delta_{a}(k = 1,2)$ and $\beta_{k}$ are the steady-state amplitudes of the intracavity modes $\hat{a}_{k}$ and $\hat{b}_{k}$, respectively. For simplicity, we assume the system to be symmetric, so the amplitudes of the two input FWS are real and equal, i.e., $\sqrt{2}\gamma_{a}\hat{a}_{1}^{\dagger} = \sqrt{2}\gamma_{b}\hat{b}_{1}^{\dagger} = \varepsilon$ and no input SH fields with $\sqrt{2}\gamma_{b}\hat{b}_{1}^{\dagger} = \sqrt{2}\gamma_{b}\hat{b}_{2}^{\dagger} = 0$. Also let the steady-state solutions in the waveguides be equal, i.e., $\alpha_{1} = \alpha_{2} = \alpha e^{i\phi_{k}}$ and $\beta_{1} = \beta_{2} = \beta e^{i\phi_{k}}$. Hence, Eqs. (6) are simplified and we can find the solutions to be

$$\beta = \frac{\kappa\alpha^{2}}{2\sqrt{\gamma_{b}^{2} + \mu_{b}^{2}}}.$$  \hspace{1cm} (7)
\[ \phi_a = \arccos \left( \frac{\alpha}{\sqrt{\gamma_a + \frac{k^2 \alpha^2 \mu_b}{2(\gamma_a + \mu_b)}}} \right), \]  
\[ \phi_b = 2\phi_a - \arctan \left( \frac{\mu_b}{\gamma_b} \right), \tag{9} \]

with \( \alpha \) being the solution of the following higher-order equation:

\[ \epsilon^2 = \alpha^2 \left[ (\gamma_a^2 + \mu_b^2) + \frac{k^2 \alpha^2 \gamma_a}{4(\gamma_a + \mu_b)} - \frac{k^2 \alpha^2 (\gamma_a \gamma_b - \mu_a \mu_b)}{(\gamma_a + \mu_b)^2} \right]. \tag{10} \]

Here, \( \mu_a = \Delta_a - J_a \) and \( \mu_b = \Delta_b - J_b \).

We utilize the standard method to derive the fluctuations of the quadratic amplitude and the quadratic phase components of the output modes. In Eqs. (3), we linearize the equations around the steady-state solutions by setting \( \delta_k = \alpha_k + \delta \alpha_k(k = 1,2) \) and \( \tilde{b}_k = \tilde{b}_k + \delta \tilde{b}_k \) and keeping the fluctuation terms only to the first order. The quantum fluctuation operators \( \delta \alpha_k \) and \( \delta \tilde{b}_k \) can be expressed as \( \delta \alpha_k = (\delta \chi_{ak} + i \delta \tilde{Y}_{ak})/2 \) and \( \delta \tilde{b}_k = (\delta \chi_{bk} + i \delta \tilde{Y}_{bk})/2 \), where \( \delta \chi \) and \( \delta \tilde{Y} \) are the fluctuations of the amplitude quadrature and phase quadrature operators, respectively. So, we obtain the linearized equations for the fluctuations as

\[ \tau \frac{dD}{dt} = MD + BD^m \tag{11} \]

with

\[ D = [\delta \chi_{a1}, \delta \chi_{a2}, \delta \chi_{b1}, \delta \chi_{b2}, \delta \tilde{Y}_{a1}, \delta \tilde{Y}_{a2}, \delta \tilde{Y}_{b1}, \delta \tilde{Y}_{b2}]^T, \]

\[ D^m = [\delta \chi_{a1}^m, \delta \chi_{a2}^m, \delta \chi_{b1}^m, \delta \chi_{b2}^m, \delta \tilde{Y}_{a1}^m, \delta \tilde{Y}_{a2}^m, \delta \tilde{Y}_{b1}^m, \delta \tilde{Y}_{b2}^m]^T. \]

\[ M = \begin{pmatrix}
-\gamma_a + p & 0 & m & 0 & -\Delta_a + q & J_a & n & 0 \\
0 & -\gamma_a + p & 0 & m & J_a & -\Delta_a + q & 0 & n \\
-m & 0 & -\gamma_b & 0 & n & 0 & -\Delta_b & J_b \\
0 & -m & 0 & -\gamma_b & 0 & n & J_a & -\Delta_b \\
\Delta_a + q & -J_a & -n & 0 & -\gamma_a - p & 0 & m & 0 \\
-n & \Delta_a + q & 0 & -m & 0 & -\gamma_a - p & 0 & m \\
0 & n & \Delta_b & -J_b & -m & 0 & -\gamma_b & 0 \\
0 & -n & -J_b & \Delta_b & -m & 0 & -\gamma_b & 0
\end{pmatrix}. \]

\[ B = \begin{pmatrix}
\sqrt{2\gamma_a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sqrt{2\gamma_a} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{2\gamma_b} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sqrt{2\gamma_b} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \sqrt{2\gamma_a} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{2\gamma_a} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2\gamma_b} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2\gamma_b}
\end{pmatrix}. \]

Here, \( m = \kappa \alpha \cos \phi_a, n = \kappa \alpha \sin \phi_a, p = k \beta \cos \phi_b, q = k \beta \sin \phi_b, \delta X_{ab(k)} \) (k = 1, 2), and \( \delta Y_{ab(k)} \) are vacuum fluctuations entering into the cavity through the coupler mirrors M1 and M2.

In order to ensure this linearization analysis to be valid, the fluctuation must be smaller than the steady-state value, which requires no positive real parts of the eigenvalues of the matrix M. Since the analytical eigenvalues of the matrix M are very difficult to obtain, we give the numerical results of the real parts of the eigenvalues for the following two cases. The first general case is \( \Delta_a \neq J_a, \Delta_b \neq J_b \), and for simplicity we assume \( \Delta_a = \Delta_b = \Delta \). The other special case is \( \Delta_a = J_a, \Delta_b = J_b \). It is well known that there is a critical value \( \epsilon_{th} \) of the input FW power for the uncoupled SHG, above which the system enters the self-pulsing regime [41]. The analytical critical value for such a coupled system is also difficult to achieve, so we define the normalized pump power of the FW as \( \sigma = \epsilon/\epsilon_{th} \). Here \( \epsilon_{th} = ((2\gamma_a + \gamma_b)/\kappa)^1/2 \gamma_a + \gamma_b \) is the threshold of a single uncoupled cavity [33]. In Fig. 2, the solid lines depict the real parts of the eigenvalues of matrix M for the general case versus the normalized pump power with \( \gamma_a = 1, \gamma_b = 2, J_a = 4, J_b = 2, \Delta = \gamma_a, \kappa = 0.01 \), respectively. From Fig. 2, one can see that the values of the solid lines are all negative in a wide range of the normalized pump power, which indicates that the system is stable and the linearization analysis is valid in this region.

For the special case, i.e., \( \Delta_a = J_a, \Delta_b = J_b \), the stationary solutions of the steady-state equations can be simplified as

\[ \beta = \kappa/2\gamma_b \tag{12} \]

with \( \alpha \) being the solution to the following equation:

\[ \epsilon - \alpha^2 - \frac{k^2 \alpha^4}{2\gamma_b} = 0. \tag{13} \]

By setting

\[ N = \sqrt{9\kappa^4 \gamma_b} + \sqrt{81 \kappa^8 \gamma_b^2 + 24 \kappa^6 \gamma_b^3}, \tag{14} \]

we find

\[ \alpha = \frac{2\gamma_a \gamma_b}{3\gamma_b^3} \frac{N}{3\gamma_b^3 \kappa}, \tag{15} \]

\[ \phi_a = \arccos \left( \frac{\alpha}{\epsilon} \left( \frac{\kappa}{\gamma_a + 2\gamma_b} \right) \right). \tag{16} \]

![Fig. 2. Real parts of the eigenvalues of the drift matrix M versus the normalized FW pump power \( \sigma = \epsilon/\epsilon_{th} \) for the general case \( \Delta_a \neq J_a \) and \( \Delta_a \neq J_b \).](image-url)
\[ \phi_0 = 2\phi_a. \]  

Although we can obtain the analytical eigenvalues of the matrix \( M \) under simplified conditions, the forms are still too complicated to be presented here. Therefore we plot the numerical results of the real parts of the eigenvalues versus the normalized pump power in Fig. 2 with \( \gamma_a = 1, \gamma_b = 2, \) \( J_a = \Delta_a = 10\gamma_a, J_b = \Delta_b = \gamma_a/4, \) and \( \kappa = 0.01, \) respectively. In Fig. 3, the solid lines represent the real parts of the eigenvalues. From this figure one can see that the critical value \( \epsilon_{th} \), the real parts of the eigenvalues of matrix \( M \) are negative, which means that the system is stable and the linearization analysis is valid in this region. Hence, the critical value \( \epsilon_{th} \) of the single uncoupled SHG also applies to the coupled system in the special case. In this work we only discuss the region in which linearization analysis is valid with the input FW values below the self-pulsing threshold.

According to the boundary condition \( D^{\text{out}} = i\sqrt{2}\gamma_j D - D^a(j = a, b) \) on the coupler mirror [37], and by using the Fourier transformation, we can obtain the output fluctuations in the frequency space

\[ D^{\text{out}} = [B(i\omega I - M)^{-1}B - I]D^p. \]

Here \( I \) is the unit matrix.

4. CHARACTERISTICS OF ENTANGLEMENT FOR THE OUTPUT MODES

Based on the sufficient inseparability criterion for CV multimode entanglement proposed by van Loock and Furusawa [36], one can sufficiently conclude that the four beams of this system are fully inseparable and posses the property of genuine quadripartite entanglement, if the following three inequalities are violated at the same time:

\[ I \quad S_1 = (\delta\langle \hat{X}_{a1} + \hat{X}_{b1} \rangle) + (\delta\langle \hat{Y}_{a1} + g_2\hat{Y}_{a2} - \hat{Y}_{b1} + g_2\hat{Y}_{b2} \rangle) \geq 4, \]

\[ II \quad S_2 = (\delta\langle \hat{X}_{a1} - \hat{X}_{b2} \rangle) + (\delta\langle g_1\hat{Y}_{a1} + g_2\hat{Y}_{a2} + \hat{Y}_{b1} + \hat{Y}_{b2} \rangle) \geq 4, \]

\[ III \quad S_3 = (\delta\langle \hat{X}_{a1} - \hat{X}_{a2} \rangle) + (\delta\langle \hat{Y}_{a1} + \hat{Y}_{a2} + g_3\hat{Y}_{b1} + g_4\hat{Y}_{b2} \rangle) \geq 4. \]

(19)

where \( g_s(s = 1, 2, 3, 4) \) are scaling factors that can be freely adjusted to minimize the correlation spectra functions for these four output beams.

In the following, we will first numerically discuss the quantum correlation spectra \( S_1, S_2, \) and \( S_3 \) with the detunings taking the values \( \Delta_a \neq J_a, \Delta_b \neq J_b, \) but \( \Delta_a = \Delta_b = \Delta. \) The quantum correlation spectra versus the analysis frequency \( \Omega = \omega\tau/\gamma_a \) are plotted in Fig. 4 with \( \epsilon = 0.6\epsilon_{th}, \gamma_a = 1, \gamma_b = 2, J_a = 10\gamma_a, J_b = \gamma_a/4, \Delta = \gamma_a, \) and \( \kappa = 0.01, \) respectively. It is obvious that the values of \( S_1, S_2, \) and \( S_3 \) are all below 4 over a quite large frequency region, which is sufficient to prove that the two FW modes and the two SH modes are genuine quadripartite entanglement. In addition, the best quadripartite entanglement is obtained at zero frequency. From this figure one can see that the correlation between the two FW modes is better than the correlation between the two SH modes. With the same parameter values, we plot the quantum correlation spectra versus the detuning normalized to \( \gamma_a \) with \( \Omega = 0 \) in Fig. 5. As this figure shows, when all four intracavity fields are on resonance, the two FW modes entangle with each other; so do the two SH modes, but these four modes are not entanglement. With an increase in the detuning, these four modes become entangled, the value of \( S_1 \) decreases to less than 4, the value of \( S_2 \) increases quickly, and the value of \( S_3 \) firstly decreases then increases, but the change is small. For the given set of parameters, the best quadripartite entanglement is achieved around \( \Delta/\gamma_a = 1.5. \)
Next we will discuss the special case, i.e., $\Delta_a = J_a$ and $\Delta_b = J_b$. In Fig. 6 we plot the quantum correlation spectra versus the normalized analysis frequency $\Omega = \omega t/\gamma_a$ for the system below the threshold value $\epsilon_{th}$ with $\epsilon = 0.3\epsilon_{th}$, $\gamma_a = 2$, $J_a = \Delta_a = 10\gamma_a$, $J_b = \Delta_b = \gamma_a/4$, and $\kappa = 0.01$, respectively. It is obvious that the values of $S_1$, $S_2$, and $S_3$ are all below 4 over a wider frequency region than the general case, which proves that these four modes are entanglement and the best quadripartite entanglement is obtained at about $\Omega = 1$. In the vicinity of $\Omega = 0$, the correlation between the two SH modes is better than the correlation between the two FW modes, but with the increase of $\Omega$, the results of the correlation are changed.

Figure 7 depicts the quantum correlation spectra versus the normalized pump power $\sigma = \epsilon/\epsilon_{th}$ with $\gamma_a = 1$, $\gamma_b = 2$, $J_a = \Delta_a = 10\gamma_a$, $J_b = \Delta_b = \gamma_a/4$, $\Omega = 1$, and $\kappa = 0.01$. In this figure, one can see that the values of these three curves are all less than 4 when $\sigma$ is greater than 0, which demonstrates that these four modes are entangled as soon as the input FWs are turned on. For the given set of parameters, the best quadripartite entanglement is achieved around $\sigma = 0.21$. Then, the degree of quadripartite entanglement decreases with the increase of $\sigma$ and the entanglement disappears at about $\sigma = 0.56$. From the curves in this figure we can also conclude that, when $\sigma$ varies from 0 to 1, the values of the correlation spectrum between the two SH modes ($S_2$ curve) and the correlation spectrum between the two FW modes ($S_3$ curve) are always below 4, which indicates that the two SH modes are entangled, and so are the two FW modes.

In Fig. 8, we plot the quantum correlation spectra versus the linear coupling parameter $J_a$ ($\Delta_a$) with $\gamma_a = 1$, $\gamma_b = 2$, $J_a = \Delta_a = \gamma_a/4$, $\Omega = 1$, $\epsilon = 0.21\epsilon_{th}$, and $\kappa = 0.01$. As shown in this figure, when $J_a$ is small, there is only quantum correlation between the FW and the SH modes of each waveguide, but these four modes are not entanglement with each other, because of the small linear interaction between two fundamental modes. With the increase of $J_a$, first the values of inequalities $S_2$ and $S_3$ decrease but $S_1$ increases, and then the values of these three inequalities are unchanged and are less than 4, which indicates that these four modes are entangled with each other as long as $J_a > 1.0$.

5. CONCLUSION

We have proposed a new scheme that can generate bright two-color quadripartite CV entangled beams based on coupled SHG processes. Two FW beams and two SH beams exit the optical cavity with spatial separation. We calculated the quantum correlation spectra of the four output modes versus different system parameters and have shown that these four modes are indeed entangled. This scheme is simple and feasible to realize in experiment and may be useful in multipartite quantum teleportation and quantum communication networks.

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